Intro to Rational Functions

9/27

Definition Function That Can be written as $f(x) = \frac{P}{Q}$ where P & Q are Polynomials

Facts/Characteristics
denominator ≠ zero
Quotient of 2 polynomials
Parent function $f(x) = 1/x$

Rationals

Examples
$\frac{3x^2y}{15xy^3}$
$\frac{2x+y}{y-3}$
$\frac{x}{x+5}$

Non-example
$\frac{4^x}{x+2}$
$\frac{|x|}{x^2+5}$

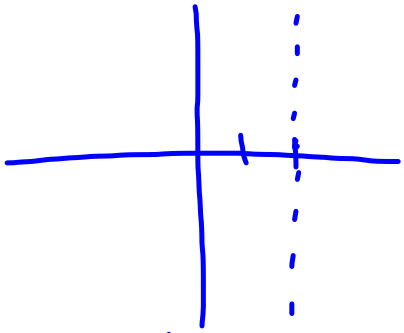
For what value is the variable undefined?

1. $\frac{4x^3y}{3t^2z}$

$t \neq 0$
$z \neq 0$

2. $\frac{3x-7}{y-2}$

$y-2=0$
$y \neq 2$



$(-\infty, 2) \cup (2, \infty)$

3. $\frac{4x^2+3x-5}{(x+6)(2x-1)}$

$x \neq -6, \frac{1}{2}$

Simplifying Rational Expressions

$$\frac{-27x^2y}{9x^4y}$$

$x \neq 0$

$y \neq 0$

$$\frac{-3}{x^2}$$

- Factor top and bottom
- Cancel like factors
- Write the answer with the restrictions on the variables

$$\frac{(x+2)}{(x+2)(x-1)}$$

$\frac{1}{x-1}$ $x \neq -2$ $x = 1$

$$\frac{4x^2-9}{4x^2+12x+9}$$

$$\frac{(2x+3)(2x-3)}{(2x+3)(2x+3)}$$

$x \neq -\frac{3}{2}$

$$\frac{x-2}{2-x} = \frac{x-2}{-1(-2+x)} = \frac{x-2}{-1(x-2)} = -1$$

$$\frac{x-5}{25-x^2} = \frac{x-5}{-1(x^2-25)}$$

$$\frac{x-5}{-1(x+5)(x-5)} = \frac{1}{-1(x+5)} = -\frac{1}{x+5}$$

Multiply

$$\frac{x^2-9}{x^2-4} \bullet \frac{x^2+4x+4}{x+3}$$

$$\frac{(x+3)(x-3)}{(x+2)(x-2)} \cdot \frac{(x+2)(x+2)}{x+3}$$

$$\frac{(x-3)(x+2)}{(x-2)}$$

$x \neq 2, -2, -3$

$$\frac{(x+3)^2}{x^2+7x+12} \bullet \frac{x+4}{x+3}$$

$$\frac{4ab^3}{4b-b^2} \bullet \frac{b^2-16}{8a}$$

$$\frac{4ab^3}{-b(4+b)} \times \frac{(b+4)(b-4)}{8a}$$

$$\frac{4ab^3}{-b(b-4)} \times \frac{(b+4)(b-4)}{8a}$$

$$= \frac{(4ab^3)(b+4)}{(-b)(8a)}$$

$$= \frac{(b^2)(b+4)}{2}$$

$$= \frac{b^3+4b^2}{2}$$

$$\frac{b^3+4b^2}{2} = \frac{b^3}{2} + \frac{4b^2}{2}$$

$$\frac{4b^3+16b^2}{8} = \frac{4(b^3+4b^2)}{8}$$

$$\frac{4b^3}{8} + \frac{16b^3}{8} \qquad \frac{b^3+4b^2}{2}$$

$$\frac{x^3-27}{x^2-9}\bullet\frac{x^2-25}{x+5}$$

$$\frac{2x^2 - 6x}{x^2 + 18x + 81} \cdot \frac{9x + 81}{x^2 - 9}$$

Divide $\dfrac{3x-6}{12x+24} \div \dfrac{x^2-5x+6}{3x^2-12}$

$\dfrac{3x-6}{12x+24} \cdot \dfrac{3x^2-12}{x^2-5x+6}$

Flip 2nd fraction

and change to multiply

$\dfrac{3(x-2)}{12(x+2)} \cdot \dfrac{3(x+2)(x-2)}{(x-2)(x-3)}$

Factor

Cancel top with bottom

$\dfrac{3(x-2)}{4(x-3)}$

Multiply

State restrictions

$$\frac{x^2}{x^2+2x+1} \div \frac{3x}{x^2-1}$$

$$\frac{3x^2-9x}{x-2} \div \frac{x^2-9}{4x-8}$$

$$\frac{4x^3}{3y^4} \div \frac{16x^2}{9y^2}$$

$$\frac{x^3-25x}{x^2-6x+5}\bullet\frac{2x^2-2}{4x^2}\div\frac{x^2+5x}{7x+7}$$

$$\frac{x(x+5)(x-5)}{(x-1)(x-5)}\cdot\frac{2(x+1)(x-1)}{4x^2}\cdot\frac{7(x+1)}{x(x+5)}$$

$$\frac{7(x+1)^2}{2x^2}$$

$$(x+1)7x+7$$

Divide

$$\frac{\dfrac{x^2-9x+14}{x^2-6x+5}}{\dfrac{x^2-8x+7}{x^2-7x+10}}$$

$$\frac{\frac{2x+10}{x-1}}{\frac{x+5}{x^2-1}}$$

$$\frac{\frac{1}{4}}{\frac{x+2}{16}}$$

Add / Subtract

MUST have a common denominator!!

$$\frac{x+1}{x+4}+\frac{x}{x+4}$$

$$\frac{2x+1}{x+4}$$

add the numerator

Keep the same denominator

$$\frac{2x-5}{x^2-1}-\frac{x-4}{x^2-1}$$

Distribute the negative

collect like terms

keep common denominator

$$\frac{6x}{3x-1} - \frac{4x}{2x+5}$$

(2x+5) (3x-1) (3x-1)

$$\frac{12x^2+30x}{(3x-1)(2x+5)} - \frac{12x^2-4x}{(3x-1)(2x+5)}$$

$$\frac{12x^2+30x-12x^2+4x}{(3x-1)(2x+5)}$$

$$\frac{34x}{(3x-1)(2x+5)}$$

$$\frac{5x+3}{3-x}+\frac{4x}{x^2-9}\cdot\frac{-1}{-1}$$

$$\frac{5x+3}{-(x-3)}+\frac{4x}{(x+3)(x-3)}$$

$$\frac{5x^2+18x+9}{-(x+3)(x-3)}+\frac{-4x}{-(x+3)(x-3)}$$

$$\frac{5x^2+14x+9}{-(x+3)(x-3)}$$

$$(x+1)(5x+9)$$

$$\frac{4}{x^2-16}+\frac{3}{x^2+8x+16}$$

$$\frac{2x+4}{x^2-x} - \frac{x+4}{x(x+1)(x-1)}$$

$$\frac{1}{2x}+\frac{5x}{x^2-1}+\frac{3}{x+1}$$

$$\frac{\frac{a^2}{9} - \frac{16}{3a}}{\frac{9}{a}}$$

$x^3 + 8$ (a = x, b = 2)

$(a+b)(a^2 - ab + b^2)$

same opposite always positive

$(x+2)(x^2 - 2x + 4)$

S O AP

$x^3 - 8$

$(x-2)(x^2+2x+4)$

S O AP

$$\frac{\frac{x-1}{4x}-\frac{1}{3}}{\frac{1}{x}-\frac{9}{x-1}}$$

$$\frac{(-x-3)(x-1)}{12(-8x+1)}$$

$$\frac{-(x+3)(x-1)}{12(-8x+1)}$$

$$12(-8x-1)$$